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MA in Philosophy

(Previous Year)

Paper- V

LOGIC

Contents:

Paper- V

PROPOSITIONAL LOGIC

Contents:

1.1 Introduction

Propositional logic is one of the fundamental branches of modern logic. Propositional logic is concerned with argument-forms whose validity depend on the connectives by which compound propositions are formed from simple ones. The propositional connectives, which are truth-functional operators, usually considered are the ones taken to correspond to 'and', 'or', 'if-then' and 'not'. Propositional logic is associated with truth-tables and truth-table tests of validity of argument-forms. An argument, as we know, consists of propositions which are either simple or compound. Propositional logic thus deals also with simple and compound statements and their relationship. It takes simple statements as a basic unit of logic. It does not analyze their inner structure of statements in terms of subject and predicate. Propositional logic, on the other hand deals with argument-forms, constituted by propositions or statements and connectives and their validity and invalidity.

Thus, this unit will analyze the various concepts, definitions and methods involved in propositional logic.

1.2 Objectives

After reading this unit you will be able to—

- *define* simple and compound statements
- *understand* what is meant by truth value of a statement
- *discuss* the nature of logical constants
- *analyse* the nature of negation, disjunction, material conditional, material disconditional statement and the rule for computing the truth values of these respectively
- *examine* inter-definability of logical constants, distinction between truth functional and non truth functional logical constants
- *sybolize* simple and compound statement
- *test* the validity and invalidity of argument forms by truth-table method
- *understand* how to construct truth table, its properties and findings

1.3 Simple and Compound Statements

Propositional logic is s two-valued logic, because it assumes that every statement has two and only two truth values. It is either true or false, but not both. So, in order to understand the basic structure of propositional logic we are required to know first the distinction between simple and compound statements and their relationships with which it deals.

All statements are of two kinds; simple and compound. A simple statement is one which does not contain another separate statement as a component part of it. For example, "Sita is a student" is a simple statement because it does not contain another statement as a component part of it. Since it does not contain another statement as a component part of it, therefore we can neither analyze nor can paraphrase it into statements. While a compound statement, on the other hand, is one which does contain another separate statement as a component part of it. Take, for example, the statement "Sita and Gita are students". This statement is a compound statement because it does contain another statement as a component part of it. Since it does contain another statement as a component part of it, therefore we can correctly analyze and paraphrase it into two separate statements, namely, "Sita is a student" and Gita is a student". Any statement which appears as a part of a larger statement is called a component of that compound statement. “Sita is a student” and “Gita is a student” are the two components of the compound statement “Sita and Gita are students”. The component parts of a compound statement may themselves be either simple or compound. Take, for example, the statement “Sita and Gita are poor and intelligent”. This is a compound statement. The components of this compound statement are also themselves compound statements.

However, it is not an easy task to determine whether or not a statement is simple or compound. Take, for example, the following these two statements; (1) “Sita and Gita are students”. (2) “Sita and Gita are sisters”. Both these statements are grammaticaly exactly alike because they have same logical structure. The first statement, no doubt, is a compound statement because it can be correctly analyzed and paraphrased into two separate simple statements, namely, “Sita is a student” and “Gita is a student” without the loss ofits meaning. But the second statement is not a compound statement because it cannot be correctly analyzed and paraphrased into two separate simple statements, namely, “Sita is a sister” and “Gita is a sister’.’ without the loss of its meaning. If we do it, we would miss their mutual relationship which the statement (2) asserts. The statement (2) asserts that Sita and Gita are sisters to one another which is not asserted by the statements “Sita is a sister” and “Gita is a sister” when we analyze it in terms of them. This clearly follows that every statement with a compound subject need not be a compound statement because not all such statements can be analyzed and paraphrased into explicitly compound statements. To say this does not mean that every statement with a simple subject is to be considered a simple statement. Every statement with a simple subject need not be a simple statement because not all such statements are simple statements. It is a component part of it. Since it does contain another separate statement as a component part of it, therefore it can be correctly analyzed and paraphrased into two separate statements, namely, “Sita is lucky” and “Sita is intelligent” without the loss of its meaning. This clearly shows that most statements with a compound subject and/ or predicate can be considered to be compound only if they can be correctly analyzed and paraphrased into separate statements which are explicitly compound.

Every statement which contains the word “not” is to be considered a compound statement because it does contain another separate statement as a component part of it. Take, for example, the statement “Sita is not a student”. This statement is a compound statement because it does contain another separate statement as a component part of it, namely, “Sita is a student”. We can correctly analyze and paraphrase it into a large statement “It is not the case that Sita is a student”. The paraphrased statement clearly shows that “Sita is not a student" is not a simple statement because it does contain another separate statement as a component part of it, namely, "Sita is a student". This clearly demonstrates that only positive statements can be considered to be simple statements. Negative statements are to be considered compound statements (except relational statements).'

The following statements are the examples of simple statements: "Ram is happy.", "Sita likes Gita.", "Married people fight a lot,", "Some people like banana with cheese.", "The man standing near by the door is a doctor". The following statements are the example of compound statements" "Sita and Gita like fish"., "Sita likes banana and apple"., "Ram thinks that Sita went to the show"., "Sunita is not a good student.", "It is not the case that Lata is brilliant student"., "Manju is either poor or intelligent.", "If Sita goes to the show, then Ram will go to the show.", "Ram will go to the show only if Sita goes to the show.", "Mohan believes that his brother is honest.", "It is possible that Rita is cheating her boy friend."

1.4 Truth Value

Every statement (simple or compound) has a truth value. It is either true or false, but not both. Truth and falsity of a statement are called the truth value of that statement. The truth value of a true statement is true and the truth value of a false statement is false. The truth value of a simple statement does not depend upon the truth values of its component part since it has no component. The truth value of a simple statement depends upon its corresponding fact, therefore it can be neither determined nor can be known by any rules of formal logic. We can know and determine its truth value only by knowing its corresponding fact on the basis of observation. But the truth value of a compound statement can be known and determined by the rules of formal logic even without knowing the fact provided that we know the truth values of its component parts because the truth value of a compound statement is nothing but a function of the truth values of its component parts of which it is composed. Any statement whose truth value depends solely upon the truth values of its component parts is called a truth- functionally compound statement.

1.5 Logical Constants

The English words such as "not", "and", "or", "if-then" and "if and only if' which we use to form compound statements are called logical constants in propositional logic. They are also called logical operators because when they operate upon a statement or statements, they always form a compound statement whose truth value depends solely upon the truth values of the statement or statements operated. Since these English words used to connect the truth values of the components of a compound statement, therefore they are also called truth-functional connectives. There are five kinds of logical constants which are used to construct the logical structure of compound statements in propositional logic. They are “not”, “and”, “or”, “if then” and “if and only if’. We can divide them into two classes : monadic and dyadic. The logical operator “not” is called a monadic operator because it requires a single statement or statement variable for its operation. Other logical operators are called dyadic or binary operators because they require two statements or statement variables for their operation. Dyadic operators are of such kind that they are always inserted between the two (simple or compound) statements unlike monadic operators. Monadic operators are always pre- fixed. They are neither inserted nor suffixed. These five truth- functional logical constants form five kinds of compound statements, namely, negation, conjunction, disjunction, material conditional and material biconditional. Since every compound statement contains logical constants in it, therefore we can say very well define it by saying that a compound statement is one which contains a logical constant in it and a simple statement in one which contains no logical constant in it. All logical constants are not fundamental because they are inter-definable in terms of one another except the logical constant “not”. But before showing how they are inter-definable, let us first discuss in detail their nature and the logical structure of different kinds of truth-functionally compound statements which result from their operation and the rules for computing their possible sets of truth values.

Conjunction:

Conjunction is a compound statement. It is formed when the logical constant ‘and’ is inserted between the two statements. In other words, when two statements are combined together by using the word ‘and’ between them, the resulting statement is called conjunction and the statements which are combined together are called conjuncts of that conjunction. Take, for example, the statement “Sita is a girl and Gita is a girl”. This statement is a statement of conjunction because it is formed from two statements when the word ‘and’ is inserted between them, namely, “Sita is a girl” and “Gita is a girl”. They are conjuncts of the conjunction “Sita is a girl and Gita is a girl”.

Use of Symbols:

It is convenient (and conventional too) to introduce a few symbols here to handle the statements of conjunction. Simple statements are conventionally abbreviated by the capital letters A, B, C, and so on. Statement variables are abbreviated by the small letters p, q, r, and s and so on. The logical constant 'and' is abbreviated by the symbol dot".". The truth values true and false are abbreviated by the capital letters T and F respectively. It is worth nothing here that the small letters p, q, r, s are not abbreviations for statements. They are abbreviations for statement variables. A statement variable is an expression which does not have any fixed meaning. It functions like blanks. It can be replaced by any statement whatsoever. When it is replaced by the abbreviations for statements, it yields statements which are either true or false. They have a fixed meaning. But statement variables are neither true nor false since they do not have any fixed meaning. Punctuations are used to aviod ambiguity and confusion when compound statements themselves are combined to form more and more complicated statements. Like mathematics, in symbolic logic too parentheses, brackets and braces are used for punctuation which we would be using in the course of discussion from time to time when the occasion demands. After introducing some few symbols, now we are in good position to translate the conjunctive statement "Sita is a girl and "Gita is a girl" in symbolic form by symbolizing its simple statements "Sita is a girl" by the capital letter S and "Gita is a girl" by the capital letter Gas S.G. The symbolic expression S.G. is a truth-functionally compound expression. Because its truth value depends solely upon the truth values of its component parts Sand G. Since the truth value of the compound expression S.G depends solely upon the truth values of Sand G, therefore we can determine and compute its truth value on the basis of the truth values of S and G by using the following rule of conjunction.

The rule of conjunction for computing the truth values of conjunction:

A conjunction is true if all its conjuncts are true. A conjunction is false if anyone of its conjuncts is false.

This rule enables us to compute the possible sets of truth values of the compound expression S.G. Since the compound expression S.G contains two statements Sand. G, and each statement has two and only two truth values, namely, true and false, therefore there are only four possible sets of truth values of the expression S.G. They are the following. If both Sand G are true, then S.G is true. If S is true and G is false, then S.G is false. If S is false and G is true, then S.G is false. If both Sand G are false, then S.G is false. These four possible sets of truth values can be expressed by means of a truth table for the logical constant"." as follows.

S	G	S.G
T	T	T
T	F	F
F	T	F
F	F	F

The same can be generalized by using statement variable p and q in the following way.

p	q	p.q
T	T	T
T	F	F
F	T	F
F	F	F

It is quite evident from the above mentioned truth table that a conjunction is true if and only if all its conjuncts are true, otherwise false.

It is important to note here that the logical constant"." admits of only one interpretation whose entire meaning is defined by the above mentioned truth table. The truth table of the conjunction p.q. states a truth value relation between p and q. It does not state any other relation between them, namely, meaning relation, temporal relation, spatial relation and so on. While its English counter terms are used in a variety of ways and hence admit more than one interpretation. Take, for example, the statement "Ramesh stepped out into the street and got hit by a car". The English word "and' occurring in it states a temporal relation between the two statements. This relation is not captured when we express it in symbolic form by using the logical constant"." The English word "and" is not the only word which we use to conjoin two separate statements together into a single compound statement. There are many other English words such as "moreover" "furthermore" "but" "yet" "however" "also" "nevertheless", "nonetheless", "although", and so on which we also use to conjoin two separate statements into a single compound statement. Even sometimes the "comma" and the "semicolon" are also used to conjoin two statements together into a single compound statement. Therefore all these English words can be translated into the dot symbol so far as truth values are concerned. What is true of the logical constant "and" and its English counter parts is also true of all other logical constants and their English counter parts, namely, "not", "or", "if-then" "if and only if'.

Negation:

Negation is a compound statement. It is formed by the use of the word ”not” on a single (simple or compound) statement putting the same thing, in other words, a negation is formed by negating a statement. For negating a statement we always use the negation sign “not”. It is worth nothing here that the negation sign is not used to combine two statements together into a single compound statement because it is a monadic operator. It is not a dyadic operator like the operator of “and” . The negation sign “not” is a monadic operator because it requires only a single statement or statement variable for its operation. It is not suffixed. It is always pre- fixed the statement negated. While other logical operators, on the other hand, are dyadic operators because they always require two statements for their operations. Take, for example, the statement “Sita is not rich”. This is a compound statement because it does contain another statement as a component part of it, namely, “Sita is rich”. Since it does contain another separate statement as a component part of it, therefore it can be correctly paraphrased into this larger statement “It is not the case that Sita is rich”. The paraphrased statement clearly shows that the logical constant “It is not the case that” is operating upon the simple statement “Sita is rich”. When it operates upon it, it produces the compound statement “It is not the case that Sita is rich”. This is a truth functionally compound statement because its truth value depends solely upon the truth values of its negated statement “Sita is rich”. The negation sign “not” is customarily symbolized by the symbol “~,, called a curl (or tilde). The negated statement “Sita is not rich”, this, can be expressed in symbolic form as “~” S. (S stands for “Sita is rich”). Its possible set oftruth values can be computed by the following rule of negation.

The rule of negation for computing the truth value of negation :

A negation is true if the statement negated is false. A negation is false if the statement negated is true. In other words, a negation simply reverses the truth value of the statement negated.

This rule enables us to compute the possible sets of truth values of the truth-functionally compound expression ~ S in the following way: If S is true, then ~ S is false. If S is false, then ~ S is true. The same can be expressed by the means of a truth table as follows:

S	~S
T	F
F	T

The same we can generalize by using statement variable p for any statement whatsoever in the following way:

p	~p
T	F
F	T

This truth table clearly shows that given any statement p :

1. If P is true, then its negation is false.
2. If p is false, then its negation is true.

Disjunction:

Disjunction is a compound statement. It is formed by using the word ”or” between two statements. Putting the same thing, in other words when two statements are combined together by inserting the word “or” between them, the resulting statement’ is called disjunction (or alternation). The statements which are combined together are called disjuncts of that disjunction. Take, for example, the statement “Sunita is poor or rich”. This statement is a statement of disjunction because it is formed by inserting the word “or” between two statements, namely, “Sunita is poor” and “Sunita is rich”. They are the two disjuncts of the disjunction “Sunita is poor or rich”.

The English word” or” is an ambiguous word. It is used in two different senses: inclusive and exclusive. Inclusive and exclusive senses are also called weak and strong senses of the word” or”. Any disjunction in which the word “or” is used in inclusive sense asserts that at least one ofthe disjuncts are true. Take, for example, the statement “Sunita is a student or Sunita is married”. This statement is a statement of inclusive disjunction because it asserts that at least one of the disjuncts is true and leaves the question open whether or not both the disjuncts are true. In legal documents the inclusive sense of “or” is expressed by the phrase “and/or” meaning that one or the other or both. Any disjunction in which the word “or’’ is used in exclusive sense asserts that only one disjunct is true, but not both. It rules out the logical possibility of both the disjuncts to be true which inclusive disjunction does not. Take, for example, the statement “Today is Monday or Tuesday”. This statement is a statement of an exclusive disjunction because it asserts that today is either Monday or Tuesday, but not both. Which means, if today is Monday, then it is not Tuesday and if today is Tuesday, then it is not Monday. It cannot be both Monday and Tuesday at the same time because the notion of Monday excludes from its meaning the notion of Tuesday and the notion of Tuesday excludes from its meaning the notion of Monday.

Both inclusive and exclusive disjunctions assert that at least one of the disjunct is true. This common meaning is the whole meaning of an inclusive disjunction and a part of the meaning of an exclusive disjunction. The symbol “v” called “vel” (or wedge or vee) is used to symbolize the common meaning of both inclusive and exclusive senses of the word “or”, namely, at least one of the disjuncts is true. The logical operator “v” is a truth functional connective because it connects truth values of any two statements p and q when it is inserted between them whose possible sets of truth values can be computed by using the following rule of disjunction.

The rule for computing the truth value of disjunction:

A disjunction is true if anyone of its disjuncts is true. A disjunction is false if all its disjuncts are false.

This rule enables us to compute all possible sets of truth values of p and q when they are disjoined together into a single compound expression. We can express it by means of a truth table as follows.

p	q	pvq
T	T	T
T	F	T
F	T	T
F	F	F

This table clearly shows that a disjunction is false if and only if all its disjuncts are false, otherwise true.

The negation of a disjunction is expressed by negating is as ~ (pvq). Sometimes the negation of disjunction is expressed by using the phrase “neither-nor” as (~p. ~q). Both symbolizations of the negation of a disjunction are correct because to deny that at least one of the two disjuncts is true is to assert at both of the disjuncts are false. The order of the words “both” and “not” is very important because it makes the difference. Take, for example, the following two statements: “Sita and Gita will not both go to the picnic” and Sita and Gita will both not go to the picnic”. Both the statement cannot be /symbolized in the same way because their logical structures differ. The former would be symbolized as ~(S.G) and the latter as ~S. ~G.

Material Conditional:

Conditional is a compound statement. It is also called hypothetical or implicative statement. It is formed by the use of the phrase “if-then”. The statement which comes between the “if’ and the “then” is called antecedent the implicant) and the statement which comes after the “then” is called consequent (the implicate). Take, for example, that statement “If Sita goes to the picnic, then Gita goes to the picnic”. This statement is a statement of conditional or hypothetical or implication because it asserts that the antecedent implies the consequent. The statement “Sita goes to the picnic” is its antecedent and the statement “Gita goes to the picnic” is its consequent. Any conditional statement formed by the use of the phrase “if-then” does not assert that its antecedent and consequent are true. It asserts only that if its antecedent is true, then its consequent is true also, that is, its antecedent implies its consequent. The general form of a conditional is “If ——then——”.

There are different kinds of conditional statement. They differ in respect to the kind of connection they express between the antecedent and the consequent. Take, for example, the following conditional statements:

1. If all humans are mortal and Raju is a human, then Raju is mortal.
2. If the figure is a triangle, then it has three sides.
3. If sugar is placed in water, then it dissolves.
4. If Sita goes to the picnic, then Gita goes to the picnic.
5. If 2+3=5, then Sri Atal Bihari Vajpayee is the Prime Minister of India.

All these conditional statements are different simply because they express different kinds of relation between their antecedent and consequent. The statement (1) is a logical conditional statement because it expresses a logical relation between its antecedent and consequent. It asserts that the consequent logically follows from the antecedent. The statement (2) is a definitional conditional statement because it expresses a definitional relation between its antecedent and consequent. It asserts that the consequent follows from the antecedent by definition. The statement (3) is a causal conditional statement because it expresses a causal relation between its antecedent and consequent. It asserts that the consequent causally follows from the antecedent. The statement (4) is a factual conditional statement because it expresses a factual relation between its antecedent and consequent. The statement (5) is a material conditional statement because it expresses a material conditional relation between its antecedent and consequent. It asserts that the consequent materially follows from the consequent.

Although these five conditional statements express different kinds of connection between the antecedent and the consequent but they all share a common meaning, namely, they all assert that if the antecedent is true then the consequent is true also. This common meaning is symbolized by the symbol "⊃" called a horseshoe (the symbol "~" also is often used). A conditional statement which expresses a common meaning shared by all conditional statements is called a material conditional statement. The horseshoe symbol "⊃" is a symbol of material implication. It captures of all other conditional statements. A material conditional statement asserts only that if its antecedent is true and its consequent is true, then the whole material conditional statement is true. Which means, it denies the conjunction of its antecedent with the negation of its consequent. So to say that p⊃q means to say that it is not the case that p is true and q is false. This can be expressed in the symbolic form as ~(p.~q). Its all possible sets of truth values can be computed by using the following rule of material conditional.

The rule for computing the truth value of material conditional:

1. Material conditional statement is true if its antecedent and consequent are true.
2. A material conditional statement is true if its consequent is true.
3. A material conditional statement true if its antecedent is false.
4. A material conditional statement is false if its antecedent is true and consequent is false.

These rules enable us to compute all possible sets of truth values for any conditional statement of the form p⊃q which can be expressed by means truth table as follows:

p	q	p⊃q	~(p. ~q)
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

This turth table clearly shows that a conditional is false only when its antecedent is true and consequent is false, otherwise true.

It is important to note here that the horseshoe symbol "⊃" is a weak symbol. It asserts only what its above truth table indicates, that is it is not the case that the antecedent is true and the consequent is false. It does not make any claim· about logical connection, or causal connection or necessary connection between the antecedent and the consequent. Implication expressed by the symbol "⊃" can be true even without there being any real connection between the antecedent and the consequent. Take, for example, the statement (5). The statement (5) does not express any real connection between the antecedent and the consequent, and yet it is materially a true statement because its antecedent and consequent are false. The truth value of material conditional statement is nothing but only a function of the truth values of the antecedent and the consequent, and not of any relation between them. This means that logical, causal, definitional and counter-factual conditional statements cannot be fully symbolized by using the horseshoe symbol "⊃".

Let us turn now to some of the English sentences which can be correctly symbolized by using the horseshoe symbol "⊃". In many cases in English we reverse the order of antecedent and consequent for stylish reason as "q, if p". The statement containing such form will be symbolized as p⊃q. There are many phrases in English which mean approximately the same as "if', such as "provided", "provided that", "supposing that", "in the event that", and so on. The sentences containing them will also be symbolized as material conditional. There is a difference between "if' and "only if'. The expression "q, if p" means p is sufficient condition for q. So it can be symbolized as p⊃q. "p only if q" means q is a necessary condition for p. So it will be symbolized as ' p⊃q ' or' ~ q ⊃ ~p '. The order of clauses in "only if' may be reversed, just as in "if' sentences. Take, for example, the statement "Only if John's dog is hungry does it bark. It will be symbolized as 'B⊃H' or ~H⊃~B. Further it is important to note that the" only if' is different from "if and only if'. The former asserts only a one-way material conditional relation while the latter asserts two-way material conditional relation. So, sentences containing the "if and only if' will be symbolized as (p⊃q). (q⊃p). There are many other words and phrases in English which are symbolized by the horseshoe symbol such as "unless". To say "p unless q" means as "p, if not q". It will be symbolized as '~p⊃q' or 'q vp'. Both these symbolizations are correct since they mean one and the same thing.

Material Biconditional Statement (or Material Equivalence):

Material biconditional statement is a compound statement. It is formed by the use of the phrase “if and only if” or “iff” when we insert it between the two statements. The word “biconditional”, as name itself indicates, expresses a two directional conditional. To say that “p if and only if q” means the conjunction of”p, if q” and “p only if q”. It can be symbolized as (p⊃q) (q⊃p). We can also symbolize it by using a triple bar symbol “≡” (often the double arrow symbol “B” is used) because they mean one and the same thing. So instead of symbolizing “p if and only if q” as (p⊃q) (q⊃p) we can symbolize it as p≡q. Both symbolizations are correct. Any English sentence which can be paraphrased in the form of “p will happen in exactly the same circumstances in what q will happen” can be symbolized by using the biconditional symbol. But in no case we can correctly use biconditional symbol for any statement of the form “p only if q” since they do not mean one and the same thing. All possible sets of truth values of any material biconditional statement can be computed by using the following rule of material biconditional statement.

The rule for computing the truth value of material biconditional statement:

A material biconditional statement is true if its both statements (between which the material biconditional sign “≡” is inserted) have the same truth value. A material biconditional statement is false if its both statements do not have the same truth values.

This rule enables us to compute all possible sets of truth values of any material biconditional statement of the form p≡q which can be expressed by means of a table as follows:

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

This truth table clearly shows that a material biconditional statement is true if and only if both statements have the same truth value, otherwise false. The material biconditional statement is also called a statement of material equivalence because the term “biconditional” is also used in the sense of material equivalence.

1.6 Inter-definability of Logical Constants

All five logical constants discussed above are inter-definable except the logical constant of negation "~". Logical constant "~" is indefinable within the system of propositional logic because it is taken as a fundamental logical constant and fundamental logical constants are indefinable by definition within the same system of logic. Logical constant "." is definable in term of "~" and "v", and "~" and "⊃" as follows. We shall use the expression "≡df' to mean "is equivalent to by definition".

$$p.q = df \sim (\sim p \vee \sim q)$$

$$p.q = df \sim (p \supset \sim q)$$

The logical constant "v" is definable in terms of "~" and ".", and "~" and "⊃" as follows:

$$p \vee q = df \sim (\sim p . \sim q)$$

$$p \vee q = df \sim p \supset q$$

The logical constant "⊃" can be defined in term of "~" and "v", and "~", and "." as follows:

$$p \supset q = df \sim p \vee q$$

$$p \supset q = df \sim (p . \sim q)$$

The logical constant "≡" is definable in terms of "⊃" and ".", and ".", "v" and"~" as follows:

$$p \equiv q = df (p \supset q). (q \supset p)$$

$$p \equiv q = df (p . q) \vee (\sim p . \sim q)$$

It is worth noting here that in each case what we mean by saying that these above mentioned expressions are equivalent by definition is that they have the same truth-table which can be verified by using the truth-table method.

Distinction between truth-functional and non-truth functional logical constants:

A logical constant is truth-functional if and only if the truth value of the compound statement which is formed by it can be completely determined by the truth values of its component parts. The important point to remember here is that it is the truth values only, and not the meanings or the relations between the statements, which determine the value of the compound. What is required to determine the truth value of compound statement is to know the truth values of the components and not their meaning relations or any other relations. All logical constants which we have discussed namely "and" "not" "or" "if-then", "if and only if' are truth functional logical constants because the truth values of compound statements which result from their operations depend solely upon the truth values of their component parts. Propositional logic is a truth functional logic because it deals with truth-functional language made up of simple statements.

A logical constant is non-truth- functional if and only if the truth value of the compound statement which it forms cannot be determined by the truth values of its component parts. Predicate logical constants "all" and "some"; modal logical constants "necessity", "possibility", impossibility", entailment"; epistemic logical constants "knowing", "believing", "thinking", and deontological constants "obligatory", "permissible" etc. are all non-truth functional logical constants because the truth value of the compound statement which they form cannot be determined by the truth values of its component parts. Take, for example, the statement "I believe that Sita is a student". The truth value of this compound statement does not depend upon the truth value of its component part, namely, "Sita is a student" because it is quite possible that Sita is not a student and yet I may believe that Sita is a student. The truth value of this compound statement does not logically follow from the truth value of its component part which is not the case with truth-functionally compound statements. The truth value of a truth-functionally compound statement logically follows from the truth values of its component parts. What is true with the non-truth- functional logical constant "I believe that" is also true with all other non-truth- functional logical constants as well mentioned above.

1.7 Symbolization of the English Simple and Compound Statements

The first thing we should identify the logical form of English sentences (or statements or propositions) while symbolizing them into symbolic form. The logical form of compound sentences can be identified by identifying heir major connectives. After identifying the logical form of compound sentences, in the next step we should identify their compound parts of which they are composed. After doing these things, in the last step we should symbolize simple sentences by using capital letters and the logical operators operating upon them by using their respective symbols. If components of compound sentences themselves are compound, we should use parentheses to avoid confusions. To identify the form of compound sentences we can use the following form listed below:

Compound Statements	**Logical Forms**
Conjunction:	(——. ——) (the dot) stands for "——— and ———"
Negation:	~ ———— (the curl) stands for "not ———"
Disjunction:	(—— v —— (the wedge) stands for "(————-or —————-)"
Conditional:	(——- ⊃ ——) (the horseshoe) stands for "If ——— then ———"
Biconditional:	(——— ≡ ——— _) (the triple bar) stands for "——— If and only if —————"

A number of examples are given below to help the readers to symbolize simple and truth- functionally compound sentences of different kinds.

Examples

English

1.	Sita is a girl.	G_S
2.	Raju likes ice cream.	Ir
3.	John is happy.	Hj
4.	Married people fight a lot.	(x) Mx ⊃ Fn)
5.	Some people like banana with cheese.	P
6.	The man standing by the door is a doctor.	D
7.	Ashok is friendly.	F_A
8.	Mohan is going to Assam.	M
9.	Sita and Gita are girls.	S.G
10.	Cats and dogs are animals.	C.D
11.	Sumi is not happy.	~S
12.	It is not the case that Jayanti is married.	~J
13.	It is false that today is Monday.	~M
14.	Mohan likes .ice and dal.	R.D
15.	It is not the case that Sita likes cats and dogs.	~(C.D)

16. Sunita is poor or rich. $P v R$

17. Sunita is neither poor nor rich. $\sim P . \sim R$ or $\sim (P v R)$

18. It is not the case that Sunita poor or rich. $\sim (P v R)$

19. If Ram goes to the show then Sita goes to the show. $R \supset S$

20. Ram goes to the show, if Sita goes to the show. $S \supset R$

21. Ram goes to the show only if Sita goes to the show. $R \supset S$

22. Ram goes to the show if and only if Sita goes to the show.
$R \equiv S$ or $(R \supset S) . (S \supset R)$

23. Ram will not go to the show unless Sita goes to the show.
$(\sim S \supset \sim R)$ or $(S v \sim R)$ or $(\sim R v S)$

24. Ram and Sita will not both go to the show. $\sim (R . S)$

25. Ram and Sita will both not go to the show. $\sim R . \sim S$

26. Unless it stops raining soon, we cannot go on picnic.
$\sim S \supset \sim P$ or $S v \sim P$

27. Only ifhe apologizes will I withdraw my legal case.
$(\sim A \supset \sim W)$ or $(A V \sim W)$ or $(W \supset A)$

28. The cat will play if she is fed, and she will not play if is not fed.
$(F \supset P) . (\sim F \supset \sim P)$ or $(F \supset P) . (P \supset F)$

29. Either Sita goes to the picnic or Gita goes to the picnic,
but Rita does not go to the picnic. $(S v G) . \sim R$

30. If Sita goes to the picnic, then if Gita goes to the picnic then Rita does not go to the picnic. $S \supset (G \supset \sim R)$

31. If Sita goes to the picnic only if Gita goes to the picnic, then Rita does not go to the picnic. $(S \supset G) \supset \sim R$

32. If Sita goes to the picnic, then Gita goes to the picnic, and if Rita goes to the picnic then Nita goes to the picnic. $(S \supset G) . (R \supset N)$

33. Sita goes to the picnic if either Gita goes to the picnic or Rita goes to the picnic. $(G v R) :: \supset > S$

34. If Sita does not go to the picnic, then it is not the case that either Gita goes to the picnic or Rita goes to the picnic. $\sim S \supset \sim(G v R)$

35. Both Sita and Gita will go to the picnic only if Rita does not go to the picnic. $(S . G) \supset \sim R$

36. It is not the case that neither Sita nor Gita will go to the picnic.(~S.~G)

37. Sita or Gita will not go to the picnic, but Rita will go to the picnic.

(~Sv~G) .R

38. Sita and Gita will not both go to the picnic but Rita and Mira will both not go to the picnic. ~ (S. G) . (~ R . ~ M)

39. If Sita does not go to the picnic then neither Gita nor Rita will go to the picnic. ~ S:: ⊃ (~G. ~ R)

40. It is not the case that either Sita or Gita goes to the picnic but neither Rita nor Mira goes to the picnic. ~(SvG).(~R.~M)

Exercises:

Symbolize the following sentences:

1. Human beings are descended from monkey.
2. Niki does not drive his car unless he gets the space.
3. Niki drives his car only if Suman is present.
4. Niki drives his car if Rekha is present.
5. Anushree is not the most athletic girl in her class, but she is the smartest.
6. Mahendra loves Sona; however, she barely tolerates him.
7. It is raining; nevertheless we will go on a picnic.
8. Sita and Gita are students, but Rita is not.
9. Neither Sushila nor Rita nor Mona will play football.
10. Sita and Gita will not both go to the picnic if Nita does not go.
11. Either Sumitra or Mira will bring the dessert, but not both.
12. Gita will prepare food if Ram and Shyam are studying.
13. Mira will not prepare unless she is happy.
14. It is not the case that John studies if and if only his mother beats him.
15. John and Mary both work late only if it is not holiday.

1.8 Testing of Validity and Invalidity of Arguments and Argument Forms by the Method of Truth Table

Validity and invalidity are purely formal characheristics of arguments and not of their contents. If any two arguments have the same logical form, then they are either valid or invalid but not both, regardless of differences in their contents. An argument is valid if its form is valid. An argument is invalid if its form is invalid. An argument form is valid if it contains no substitution instance with true premises and a false conclusion. An argument form is invalid if it contains at least one sub situation instance with true premises and a false conclusion. By the "form" of an argument we mean the general pattern or structure of argument which has no definite or particular meaning or truth value. Argument forms contain statements variables and statement variables are different from statement constants. A statement constant is an expression that has a definite meaning or truth value. A statement variable is an expression that has no definite meaning or truth value. It can take any statement constant as its value. In propositional logic statement constants and statement variables are used instead of numerical constants and numerical variables which are used in mathematics. A statement constants is a capital letter used as an abbreviation for a simple statement (or proposition of sentence). It is a simple sentential or statement constant. A statement variable is a lower case letter from the middle of the alphabet which stands for any particular substitution instance or simple statement. It is a simple sentential or statement form. A statement form is a well formed formula (simple or compound) which contains statement variables such that when statement variables are replaced by statements (same statement replacing same variable throughout), it yields statements. A substitution instance of a statement form is a statement which is obtained from the form of that statement by substituting uniformly some statement, simple or compound, for each variable in the form. It is important to note here that all the statement constants and substitution instances have definite meanings and truth values. That is why they cannot be replaced by any other statement constants and substitution instances. The statement variables and forms have no definite meanings and truth values. That is why they can be replaced by any other statements or forms.

The following table illustrates the various sentential or propositional elements which we have described above.

Forms	Instances
Statement variables	Statement constants
p, q, r, s,	A,B,C,D
Statement forms	Statements

(p ⊃ q). (p v q) . ~ r	(A ⊃ B) . (A v B) . ~ R
(p. q) v (~p. ~ q)	(A. B) v (~A. ~ B)

An argument form is a group of statement forms. All its substitution instances are arguments. For example, all substitution instances of this argument form

P ⊃ q

p   / :. q

are arguments. For example,

A ⊃ B

A   /:. B

and   C ⊃ D

C   /:. D

1.9 Truth Table Construction

To test validity or invalidity of any given argument form, we will have to examine all its possible substitution instances in order to know whether or not it has substitution instances with true premises and false conclusions. But this is impossible to know because an argument form has infinite number of substitution instances. To say this does not mean that there is no method to test validity or invalidity of any argument form. The logicians have discovered a method, called a truth table method, by which we can test whether or not any argument form has a substitution instance with true premises and a false conclusion. The procedure or rule for constructing a truth table is as follows.

1. Count the number of different statement variables occurring in premises and conclusion of any given argument form and write down them at the top of the table in alphabetical order to the left.
2. Count the number of different component parts (simple and compound) occurring in premises and conclusion of that given argument form and write them at the top right to the symbols of statement variables in order from simple to more complex.
3. Count the number of statements of the different premises and conclusion and write them at the top right to the expressions of their component parts.
4. Workout the possible sets of truth values of different statement variables occurring in premises and conclusion and write them in the rows.

5. The rows are drawn by horizontal lines and the columns by the vertical lines. The rows represent possible sets of substitution instances and column represents the possible combination of truth values for the statement variables and their combinations occurring in the premises and conclusion.

6. Number of the rows depends upon the number of different statement variables. Since every single statement variable has only two possible truth values, therefore its substitution instances also will have only two possible truth values. It may be either true or false. For working out the number of possible sets of combination for any number of statement variables, we can use the formula 2n. For one statement variable, it will be 2; for two different statement variables, it will be 2 x 2; for three different statement variables, it will be 2 x2 x 2 and for four statement variables, it will be 2 x 2 x 2 x 2 and so on. In general we can, thus, say for n different statement variables in a statement or an argument form, the number of possible sets of truth values for the statement variable will be 2^{n} (and not n).

7. The order of arrangement in columns of the truth values of each of the variable would be as follows: where n is the total number of variables, the column under the mth variable contains 2^{m-l} sets of 2 n-m T,s followed by 2^{n-m} F,s. For example, if we have a compound statement of two variables, then there will be 22 rows and the column under the first variable will have 2^{1-1} or 2°, that is, one set of2^{2-1} or two T's followed by the same number of F's, that is, TTFF. If we have a compound statement of four variables then there will be 2^{4} or sixteen rows in the truth table and the column under the third variable will have 2^{3-1} or four sets of 2^{4-3} or two T's followed by the same number of F's.

8. When we evaluate any truth-functionally compound statement by constructing its truth table, we determine its characteristic matrix-number. For example, the matrix number of p v q is TTTF and the matrix number of p.(q v r) is TTTFFFFF.

After constructing a truth table for any argument form, the only thing which is left to do is to check whether or not it is valid by using the rule of validity. If it has no substitution instance with true premises and a false conclusion, then it is valid. But if it has a substitution instance with true premises and a false conclusion, then it is invalid. In other words, if there is no row in the truth table of that argument form in which premises are true and conclusion is false, then it is valid. But if there is a row in the truth table in which premises are true and conclusion is false, then it is invalid. An argument is valid if it is an instance of an valid. argument form. It is invalid if it is an instance of an invalid argument form. Take, for example, the following argument form :

p ⊃ q

~q   /:.~p

The truth table for this argument form is an follows:

p	q	~q	~p	p ⊃ q
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

This argument form is a valid argument form because it has no substitution with true premises and a false conclusion. In the table wherever all the premises are true, its conclusion is also true (see the last row and columns third fourth and fifth).

Take another argument form :

p ⊃ ~ q

p   / :. ~ q

The truth table for this argument form is as follows:

p	q	p ⊃ q	~q
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	T

This argument form is invalid because it does contain in the first row in the truth table in which its premises are true and conclusion is false.

Exercises:

a. Use truth tables to decide the validity or invalidity of the following argument forms:

l.	$p \supset q$ p :. q	6.	$p v q$ $\sim p$:. q
2.	$p \supset q$ q :. p	7.	$p \supset q$ $q \supset p$:. $p \vee q$
3.	$p \supset q$ $\sim q$:. $\sim p$	8.	$p \supset q$ $q \supset r$:. $p \supset r$
4.	$(p \supset q) . (r \supset s)$ $p \vee r$:. $q \vee s$	9.	$\sim(p v q)$:. $\sim p . \sim q$
5.	$(p \supset q) . (r \supset s)$ $\sim q \vee \sim s$:. $\sim p \vee \sim r$	10.	$(p \vee q) \supset (p . q)$ $\sim(p v q)$:. $\sim (p . q)$

b. Use truth table method to decide whether or not the following arguments are valid after symbolizing them:

1. If Sita is elected class president, then either Gita is elected secretary or Rita is elected treasurer. Sita is elected class president. Therefore, Either Gita is elected secretary or Rita is elected treasurer.
2. If Sita is elected class president, then neither Gita is elected secretary nor is Rita elected treasurer. Either Gita is elected secretary or Rita is elected treasurer. Therefore, Sita is elected president.
3. If weather is warm and the sky is clear, then we will go to picnic and we will enjoy. It is not the case that if the sky is clear, then we will enjoy. Therefore the weather is not warm.

4. If the Prime Minister resigns, the party will split and there will be an election. If there is election, nation situation will not improve. The nation situation will improve. Therefore the Prime Minister will not resign.
5. If A is elected, then B will resign. If C is elected, then B will not resign. Therefore if A is elected, then C will not be elected.

Statement Forms :

A statement form is one that contains statement variables, such that when all its statement variables are replaced by statements - the same statement replacing the same statement variable throughout - the resulting expression is a statement. Take, for example, the expression $p \supset (q \supset r)$. This expression is an expression of statement form because it contains statement variables p, q, r. When we substitute statements A, B, C, for p, q, r respectively, we find the expression $A \supset (B \supset C)$ which is an expression of a compound statement. It is important to note that any statement can be substituted for statement variables. The statement which is substituted for the statement variable is said to have the same form or to be an instance of it. Since the expression $A \supset (B \supset C)$ results from $p \supset (q \supset r)$ when we substitute A, B, C for statement variables p, q, r, therefore it is called a substitution instance of it. There are three kinds of statement form: tautologies, contradiction and contingent.

Tautologies:

A statement form that contains only true substitution instances is called tautologous or tautology. Since tautologies contain only true substitution instances, therefore they can never be false. They are true under all interpretations and circumstances, regardless of their contents. A statement is tautology if and only if it is a substitution instance of a tautology. Take, for example, the following statement form p v ~ P. This statement form is a tautology because it has no false substitution instance. It has only true substitution instances under the major operator "v" which is quite evident from the following truth table of p v ~ p.

P	~p	pv~p
T	F	T
F	T	T

This table clearly shows that there is no instance in which the truth value under the major operator is false. Hence it is a tautology. All its substitution instances are tautologous statements, for example, B v ~ B.

Contradiction:

A statement form that contains only false substitution instances is called contradiction or contradictory. Since contradiction always contains only false substitution instances, therefore it can never be true. It is false under all interpretations and circumstances, regardless of its content. Take, for example, the statement form p . ~ p. This statement form is a contradiction because it has no true substitution instance. It has only false substitution instances which is quite evident from the following truth table.

P	~p	p.~p
T	F	F
F	T	F

The table clearly shows that there is no substitution instance in which the truth value under the major operator is true. Hence it is a contradiction. A statement is contradictory if and only if it is a substitution instance of a contradiction. For example, the statement B . ~ B.

Contingent:

A statement form that is neither tautology nor contradiction is called contingent. It contains both true and false values under the major operator. Take, for example, the expression p v q. This expression is an expression of contingent because it has both true and false substitution instances under the major operator “v” which is quite evident from the following truth table.

P	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

This table clearly shows that it has both true and false substitution instances. Hence it is a contingent. A statement is contingent if and only if it is an instance of contingent. For example, statement B v C is a contingent statement because it is an instance of p v q.

It is quite evident from the above discussions that the negation of a tautology is a contradiction and the negation of a contradiction is a tautology. But the negation of a contingent is a contingent. The truth of a tautology is called a logical truth because it is true by the rule of formal logic. The falsehood of a contradiction is called a logical falsehood because it is false by the rule of formal logic. Hence the logical truth of tautology and logical falsehood of contradiction can be known on a priori ground independent of empirical facts just by knowing their forms. But the truth of contingent cannot be known on an a priori ground because it depends upon the empirical fact. It can be known only by observation. Tautologies and contradictions played little role in our ordinary language because they do not give any information about the empirical facts. But they do play a very important role in logic simply because they do not make any claim about the empirical world. Their truth value is independent of circumstances no matter what the worldly facts are. All scientific statements are contingent statements. They cannot be proved true or false by any rules of formal logic.

There is a connection between the concepts oflogical equivalence and tautology. Two statements are said to be logically equivalent if and only if the result of joining them together with biconditional is a tautology. Take, for example, the following two expressions. : p and ~ ~ p. Both expressions are logically quivalent because the result of joining them together with the biconditional is a tautology which we can see from the following truth table.

p	~~p	p ≡ ~~p
T	T	T
F	F	T

Both the expressions are not only logically equivalent but also logically imply each other. To say this does not mean that both the expressions are one and the same. There is a difference between the two. The concept of logical implication is not a symmetrical relation. It is one directional relation. So to say that one form logically implies another does not necessarily mean the second also implies the first. It may or may not imply. While logical equivalence is a symmetrical relation because to say that one form is logically equivalent to another is to say that they logically imply each other. Two statements are said to be logically equivalent if and only if they logically imply each other or have identical truth values. Since logically equivalent statements logically imply each other, therefore we can validly infer one from the other.

Exercises:

a. Use the truth table method to decide whether the following statement forms are tautologies, contradictions or contingents:

1. q ⊃ (q ⊃ p)
2. (p v q) . (~ p v ~ q)
3. p ⊃ (p ⊃ p)
4. (p ⊃ p) ⊃ p
5. ~(p. ~q).q
6. (p . q). p
7. [(p ⊃ q) ⊃ q] ⊃ q
8. [(p ⊃ q) ⊃ P] ⊃ P
9. [(vp. p) ⊃ q] V P
10. (~ P v ~ q). (q ⊃ p)

b. Use truth table method to decide whether or not the following expressions are logical equivalences:

1. (p ⊃ q) ≡ (~q ⊃ ~ p)
2. (p ⊃ q) ≡ (~ p v q)
3. [(p . q) ⊃ r] ≡ [p ⊃ (q ⊃ r)]
4. [p ⊃ (q ⊃ r)] ≡ [(p ⊃ q) ⊃ r]
5. (p v q) ≡ (q v p)
6. ~ (p . q) ≡ (~p v ~ q)
7. [(p v q) ⊃ p] ≡ [~ q ⊃ ~ (p . q)]
8. [(p v q) . (p v r)] ≡ [(p . q) v (p . r)]
9. ~(p.q) ≡ (~pv~q)
10. (p ≡ q) ≡ [(p . q) v (~p . ~ q)]

c. Symbolize the sentences and then test their forms to determine whether they are tautologies, contradictory, contingent, or equivalent.

1. If it is not Monday then we will go to school, but if it is Tuesday, we will not go to school.
2. Manoj will go to school if and only if it is Friday.

3. If Sita does not go on picnic, it is not the case that either Gita or Rita will go on picnic.

4. Manju wins her first game only if either John or Mary does not win its first game.

5. The Government will fall or they will get a vote of confidence and stay empower.

6. If A is elected, B will resign and if A is not elected, B will not resign.

7. Postal rates will increase only if the number of postal workers is not reduced and their salaries increase.

8. There will be neither peace nor war.

9. If there is no increase in production and decrease in interest rates, there will be increase in unemployment.

10. As long as the country does not get external threats, there will be no war.

1.10 Summing Up

After going through this unit you have learnt that propositional topic is concerned with simple and compound proposition with two truth values. It also analyses the concept of truth function truth table and different logical constents and variables and its uses to prove the truth value of an argument. You are again clear here how to construct truth table to test the truth value of simple and compound proposition. This unit will help you to go depth of the object for further study.

1.11 References/Suggested Readings

Ambrose and Lazerowitz	:	Fundamentals of Symbolic Logic
P. Suppes	:	Introduction to Logic (Part II on "Set Theory")
Copy	:	Symbolic Logic
Jeffry	:	Formal Logic: Its scope and Limits

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